

EXPLORING ALGEBRA THROUGH PATTERNS

*STUDENTS USE ALGEBRAIC REASONING TO UNDERSTAND
MATHEMATICAL PATTERNS*

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SUITABLE FOR AGE(S)

10-16 years

SUBJECT(S)

Mathematics

KEY FOCUS

Algebraic thinking

INTRODUCTION

Algebra is an essential tool in mathematics and physics. Students use algebraic skills to make generalisations and explore patterns in the world around them. Algebraic thinking begins at an early age, when young children represent mathematical operations with objects, their bodies (e.g. finger counting), images, drawings, and sounds. As children get older, they start to make deeper connections between algebra and formal arithmetic operations.

Algebraic thinking involves noticing patterns and forming generalisations and is essential to support learners to make connections to real life. It is suggested that algebraic thinking involves three main strands (Blanton 2008):

- The study of structure in the number system
- The study of patterns relationships, and functions
- The process of mathematical modelling

Algebra is often referred to as generalised arithmetic (van de Walle). To support students in generalising operations or patterns, they need opportunities to explore multiple examples and time to discuss what they observe, with particular emphasis on analysing the structure of what they are seeing.

In this task, students are presented with a challenge: they must determine the missing number in the patterns presented. Depending on the learning context, additional structures and supports can be provided through teacher questioning.

TASK DESCRIPTION

This rich mathematical task offers students the opportunity to reason about patterns and can serve as either an introduction to algebra through patterns or as a means to explore systems of equations.

The task is designed to be accessible for all learners aged 10–16 and is applicable across various stages of mathematics learning journey. The task is presented as an open task, though more structure can be introduced through questioning. It can be used as a way for younger learners to explore patterns and for older, more experienced learners to reason about solving simultaneous equations.

Students work collaboratively to explore visual and numerical patterns. They engage in problem-solving activities that develop their ability to generalise patterns and connect them to algebraic expressions.

TASK PREPARATION

The teacher prepares selected pattern images and related tasks. Students work in groups and are encouraged to approach the task creatively and provide a clear explanation of their reasoning.

Materials needed:

- Projector or screen to display images of the patterns
- Printed copies of the pattern task, if desired
- Flipcharts or student notebooks for recording strategies and solutions.

TASK IMPLEMENTATION

Step 1: The Warm-Up

The task begins by encouraging students to think about patterns. The teacher presents an image (Figure 1) to the class and asks them to memorise the code for each number — students are given 1 minute. The image is then removed from the screen, and students are asked if they can describe the patterns for the numbers. It is important at this point to ask them how they memorised the pattern and to provide opportunities for the class to share ideas.

Can you memorise these codes for numbers 1-9?

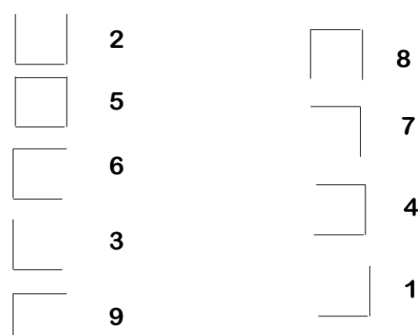


Figure 1. Warm-up activity

Following the discussion, the image is presented again in a different format (Figure 2), and students are invited to reflect on how changing the format affects their interpretation.

One simple change in presentation



Figure 2. Change in presentation

This generates a discussion about how we observe and interpret patterns, as well as how different representations can impact memory. Discuss with students how the way that we look at things sometimes changes how we interpret patterns and remember information. This warm-up provides opportunities for students to examine patterns in different ways and share their interpretations of what they see when looking at mathematical images.

Step 2: Find the Missing Number

Main Task: Students work in pairs to solve the main challenge: Find the missing number in a presented set of patterns (Figure 3).

Find the number which will replace ? In the figure

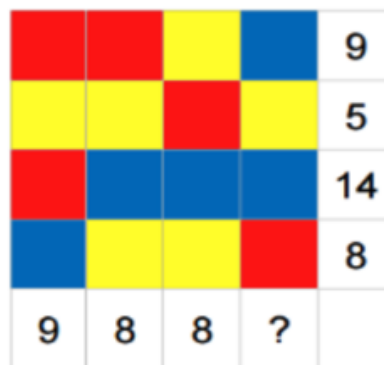


Figure 3. Task - Find the missing number*

Recommended structure:

Students work in pairs as they work through the task. It is recommended that students be given some time (approximately 10 minutes) to work in their pairs without interruption. Students should be encouraged to record their ideas and prepare to share their initial thoughts with the whole class.

After the initial pair work, a whole-class discussion is held where students present things they noticed. If a pair has already found the missing number, they are challenged to find another way to justify their answer.

The primary objective of the activity is for students to describe how they found the missing number. When students have found the number, they should be able to justify and explain their reasoning. They should also be encouraged to explore alternative ways to explain the answer they arrive at [Some possible solutions to the task are provided in the Appendix].

This task can be implemented with class groups at many levels. Some suggested teacher questions and approaches are included below:

➤ Upper Primary

At this level, the focus is on exploring patterns and developing algebraic thinking. Support students with questions such as:

- What could each of the colours represent?
- Why is there a different number at the end (and bottom) of each row?
- Do any rows or columns look similar?
- Can you tell (or write down) a statement explaining what each of the rows means?

➤ Lower Secondary

Each of the questions above will also help lower secondary students to think algebraically. Additionally, they can be encouraged to assign a letter to each of the colours. Students can also be guided to consider combinations of rows or view the square as a whole (e.g., Can you tell me something about the 'value' of rows 3 and 4 together? Can you tell me something about the 'value' of the square as a whole?).

➤ Moving Towards Upper Secondary

As students progress to the upper secondary level, they can be encouraged to make connections between this representation and solving systems of equations (e.g., solution B in the Appendix). For example, they can derive equations from the pattern and solve them algebraically. Students can also be challenged to create their own pattern squares with two or three variables

KEY LEARNINGS

This task has been used with pre-service teachers in Ireland (both primary and secondary levels) and in a workshop with STAMPed participating teachers at the final STAMPed conference. Teachers commented on the flexibility of the task and its potential to help learners understand what is happening when solving simultaneous equations.

CONCLUSION

Exploring patterns provides an effective pathway to developing algebraic thinking. Students gain confidence in working with variables and expressions through collaborative exploration and reflection. The task is versatile and can support key curricular transitions — particularly moving from informal pattern work to formal algebra.

REFERENCES

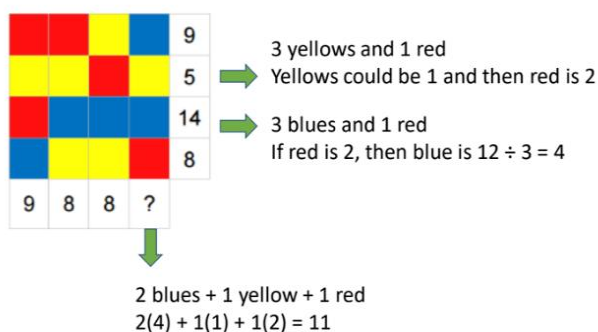
- Van de Walle, J. A., Karp, K. S., & Bay-Williams, J. M. (2014). Elementary and Middle School Mathematics. Pearson.
- Boaler, J. (2015). Mathematical Mindsets: Unleashing Students' Potential through Creative Math, Inspiring Messages and Innovative Teaching. Jossey-Bass.

Note: *Original source of the "find the missing number" image is unknown. It has been used in another Erasmus+ funded project: SkillUp Game.

APPENDIX

Possible Solutions

Solution A



Solution B

A set of equations, for example:

				9
				5
				14
				8
9	8	8	?	

$$\left\{ \begin{array}{l} 2x + y + z = 9 \\ 3y + x = 5 \\ x + 3z = 14 \end{array} \right.$$

Solution C

				9
				5
				14
				8
9	8	8	?	

$$\left. \right\} = 36$$

$? = 36 - 25$
 $? = 11$

Solution D

				9
				5
				14
				8
9	8	8	?	

↓
11

				9
				5
				14
				8
9	8	8	?	

→ 22