EGGS, APPLES AND CENTIMETRES - TRANSFORMATION OF UNITS

STUDENTS STUDY CONCRETE MATERIALS TO MEASURE 1D, 2D & 3D QUANTITIES

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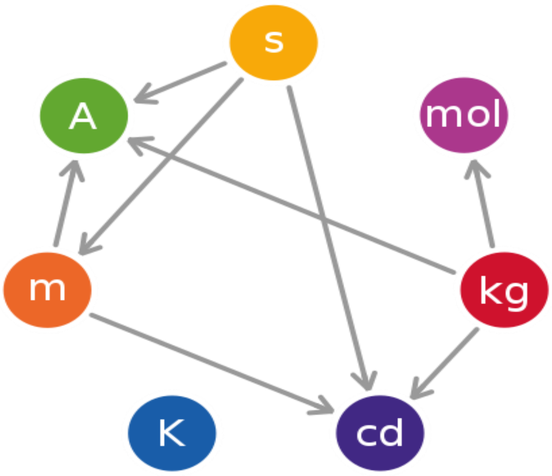

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SUITABLE FOR AGE(S)	SUBJECT(S)	KEY FOCUS
10- 18 years	Physics, Mathematics	Transforming units

INTRODUCTION

This task provides students with a practical opportunity to deepen their understanding of units and unit transformations in mathematics and physics. It highlights the importance of using correct units, introduces SI prefixes, and helps students gain confidence in converting 1D, 2D, and 3D measurements using unit prefixes and base units. Through guided inquiry and collaborative activities, students develop the essential skills needed for correct and automatic handling of units in science and mathematics problem-solving.

Units are introduced into the school curricula for the first time during mathematics lessons on measuring distance. Later, in mathematics lessons, students also learn about units for area and volume. A proper introduction to these concepts in math should help students (1) understand why units are essential in all measurements and calculations in physics and (2) develop automatic skills in converting units using prefixes. They should also realise that complex units can be broken down into a few basic units.

Teachers often report that many students struggle to understand the meaning of units and find it challenging to develop the skills needed to convert units, both using unit prefixes and base units. In early mathematics lessons, students initially perform calculations using only numbers. When dealing with simple measurements of distance, time, temperature, and other quantities, students often overlook units when presenting their results or solving word problems. Therefore, it is essential that, from the beginning, both mathematics and science teachers emphasise the importance of including units whenever appropriate. The task is designed to address these issues in a few steps that can be applied in both mathematics and physics lessons.

TASK DESCRIPTION

The task consists of two parts:

The first part is a brief inquiry activity that involves measuring the dimensions of objects found in the classroom. The second part begins with building real paper objects and utilising them in 1D, 2D, and 3D environments. The subsequent step in the second part involves calculations using unit prefixes in 1D, 2D, and 3D, as well as other units.

- **Part 1: A structured inquiry-based learning (IBL)** activity in which students explore measurements using standard tools, practice using unit prefixes (milli-, centi-, deci-), and develop correct multiplication notations for area and volume.
- **Part 2: A group problem-solving task** in which students construct and fill large squares and cubes to reinforce spatial understanding of unit powers. They then

use these physical models to perform unit transformations across dimensions (e.g., from cm^2 to m^2, or from cm^3 to dm^3).

This task can be implemented regularly, initially in mathematics classes and later in physics classes, until students develop their habits of (1) using units in all calculations related to measurements, (2) correctly transforming units with the use of unit prefixes, and (3) accurately transforming the powers of units with the use of unit prefixes. The task is suitable for on-site classes. The second part can be taught in person or adapted for remote learning, utilising breakout rooms.

The two parts can be used on different occasions:

➢ Part 1 – as an introduction to distance, area, and volume measurements in mathematics

➢ Part 2 – as an introduction to the transformation of units in 1D, 2D, and 3D in mathematics classes and in physics classes

➢ Part 2 (points 4-7) – repeatedly, on different occasions in physics and mathematics classes, as an algebra exercise building the automatism of proper transformation of units with the use of unit prefixes.

TASK PREPARATION

A prerequisite for completing the task is that students understand simple fractions and decimal fractions, as well as how to multiply and divide them. The task is organised mostly as group work. However, all students need to take their own notes and do their own calculations at several moments during the implementation.

Materials needed:

For part 1:

- Rulers, meter sticks
- Measuring tools marked in mm, cm, dm
- Regular classroom objects (boxes, books, etc.)
- Worksheet with units and prefixes (dm, cm, mm, mA, mJ, etc.)
- Scissors, glue, coloured paper for making squares and cube templates

For part 2:

- Pre-cut squares (1 cm^2 and 1 dm^2) and cubes (1 cm^3 and 1 dm^3) made of sturdy paper (200 g/m^2 or more)
- Materials for building cube and square shells (wooden or plastic sticks, wires, or similar)
- Additional worksheets with calculations involving SI units and prefixes
- Digital collaboration tools (e.g., Jamboard, Google Drive, OneNote Miro, Mural, etc.)) if online.

TASK IMPLEMENTATION

Part 1 – Structured Inquiry Activity

Part one is designed as an IBL activity, with some components completed by the whole class and others by groups.

1. At the beginning of this Part, an introductory story (e.g., Appendix 1) is used to prompt students to reflect on the necessity of units in communicating measurement or calculation results in science.

2. In groups, students learn about 1 meter (m), 1 centimetre (cm), and 1 millimetre (mm) by examining various distance measuring tools. A measuring tool with a scale in millimetres (mm) or decimetres (dm) should also be prepared.

3. The teacher summarises:

> Prefix d stands for $\frac{1}{10}$ or in other words 0,1 and reads "deci"
>
> Prefix c stands for $\frac{1}{100}$ or in other words, 0,01 and reads "centi"
>
> Prefix m stands for $\frac{1}{1000}$ or in other words, 0,001, and reads "milli"

4. Students are provided with a worksheet that includes various units with the prefixes mentioned above (such as dm, cm, mm, ms, g, mA, cV, and mJ), and are instructed to perform calculations and express the results without using these prefixes in groups.

5. In groups, students measure the dimensions of various classroom objects (length, width, and height), record their measurements with appropriate units, and compare their results. They note their conclusions.

6. Students cut out three squares of different sizes, preferably from papers of different colours, and all students in each group measure the width and length of all squares, recording their measurements with units. Students are asked to measure the dimensions in centimetres and millimetres. They compare their measurements in different units and work together to draw a conclusion. The teacher reviews the conclusions.

7. With the use of three patterns prepared by the teacher, students cut out the 2D shape of a cube and assemble the cube following the instructions given by the teacher. Students are asked to measure the dimensions of the cube in both centimetres (cm) and millimetres (mm). Students compare their own measurements in different units and formulate the conclusion in a group. The teacher checks the conclusions.

8. Students assemble a cube from pre-cut 2D shapes, following the instructions provided by the teacher, and measure its dimensions in centimetres (cm) and millimetres (mm). They compare their measurements in different units and draw a conclusion as a group. The teacher reviews the conclusions.

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9. The teacher introduces multiplication notation:

> $1\,m$ means $1 \cdot m$ (one time a metre)
>
> $1\,dm$ means $1 \cdot dm$ (one time a decimetre)
>
> $1\,cm$ means $1 \cdot cm$ (one time a centimetre)

10. The teacher instructs students to convert all their notes on dimensions, such as "3 cm long, 4 cm wide, 1 cm high," into a notation that uses multiplication.

11. The teacher introduces the brief notation for area units.

> The area of a square 1 m long and 1 m wide is shortly written as
>
> $1\,m \cdot 1\,m = 1\,m^2$
>
> The area of a square 1 dm long and 1 dm wide is shortly written as
>
> $1\,dm \cdot 1\,dm = 1\,dm^2$
>
> The area of a square 1 cm long and 1 cm wide is shortly written as
>
> $1\,cm \cdot 1\,cm = 1\,cm \cdot 1\,cm = 1\,cm^2$

12. The teacher asks students to express the areas they measured in the structured IBL in short notation (without unit transformation).

> The volume of a cube 1 m long, 1 m wide and 1 m high is shortly written as
>
> $1\,m \cdot 1\,m \cdot 1m = 1\,m^3$
>
> The volume of a cube 1 dm long, 1 dm wide and 1 dm high is shortly written as
>
> $1\,dm \cdot 1\,dm \cdot 1\,dm = 1\,dm^3$
>
> The volume of a cube 1 cm long, 1 cm wide and 1 cm high is shortly written as
>
> $1\,cm \cdot 1\,cm \cdot 1\,cm = 1\,cm^3$

13. Students write their recorded dimensions and the volumes they measured in the structured IBL (without transformation of units).

Part 2 – Group Problem Solving and Unit Transformations (In-class or Remote)

Preparation for the activity: Before the class, students are asked to make and bring squares and cubes cut out of thick paper. They need a total of 100 squares measuring 1 dm by 1 dm and 100 squares measuring 1 cm by 1 cm; at least 100 cubes measuring 1 dm by 1 dm by 1 dm and at least 100 cubes measuring 1 cm by 1 cm by 1 cm. Ideally, if several classes work together, students could make 1000 cubes of each size.

Steps:

In the class, students build two shells: a square of dimensions 1 m x 1 m and a square of dimensions 10 cm x 10 cm.

Students first fill the shells with squares, counting the squares as they place them and writing notes on the results in a sequence:

- First, the larger shell with small squares measuring 1 cm x 1 cm,
- Second, the smaller shell with 1 cm x 1 cm squares,
- Third, the larger shell has 1 dm x 1 dm squares.

In groups, students compare their results and record the conclusions. These conclusions are then shared and discussed with the whole class.

The teacher reinforces multiplication rules for area and volume, and recaps SI prefix relationships and writes on the blackboard:

The area of a square 1 m long and 1 m wide is shortly written as

$$1\,m \cdot 1\,m = 1\,m^2$$

The area of a square 1 cm long and 1 cm wide is shortly written as

$$1\,cm \cdot 1\,cm = 1\,cm^2$$

The area of a square 10 cm long and 10 cm wide is shortly written as

$$10\,cm \cdot 10\,cm = 1\,dm \cdot 1\,dm = 1\,dm^2$$

$1\,m$ means $1 \cdot m$ (one time meter)

$1\,dm$ means $1 \cdot dm$ (one time decimeter)

1 dm = 0.1 m so yellow parts can be replaced in calculations

1 m = 10 dm so yellow parts can be replaced in calculations

Students are asked to recalculate all their measurements from PART 1 and express areas in mm^2, cm^2, dm^2, m^2, and volumes in mm^3, cm^3, dm^3, m^3. Students come back to their notes from the structured IBL experiment.

The teacher provides a worksheet for other calculations using the transformation of unit prefixes related to dimension units. In physics classes, (also recommended in mathematics classes) the teacher provides a worksheet for calculations that involves the transformation of unit prefixes in relation to other units (kg, km/s, N, Pa, J, A, V) to ensure that students do not attribute the prefixes d, c, and m solely to the dimension units. If the units 'kg' and 'km' are included, the prefix k needs to be explained to students.

KEY LEARNINGS

This task was partially implemented in professional learning communities of novice teachers. Physics teachers identified the transformation of units as the most challenging aspect of students' calculations during problem-solving activities in physics classes.

In teachers' opinions:

- Part 1 of the task should be implemented in mathematics classes and repeated at least a few times. It could be done once more during the first lessons of physics.
- When mathematics teachers introduce units, they should mention units other than only dimensional units, and this can be done through the implementation of PART 1.
- Part 2 addresses the transition problem from mathematics to physics. The use of multiplication notation and colours for the replacement of some units by other units is highly recommended for a long time since its first introduction.

Workshop participants agreed that they could find various opportunities to implement the module in their teaching practice at the primary school level (Part 1) and the lower secondary school level (Part 2, points 4-7). They frequently utilise different exemplary worksheets to develop students' automation in transforming units, using unit prefixes, and manipulating powers of units. Many reported that they already incorporate elements of this module into their current teaching practice, but would be pleased to approach the topic more comprehensively as presented in this learning unit.

CONCLUSION

This rich task serves as a bridge between mathematics and physics, helping students form a deep and transferable understanding of units and measurements. When implemented regularly and across both subjects, it encourages long-term habit formation for proper unit use, understanding of prefixes, and dimensional reasoning.

APPENDIX

A. Below, you will find a short story. Read it carefully and answer the questions below the text in groups.

It took me 30 to move 5 from the grocery to my house. During this time I climbed up-hill 50. I got quite tired since, in addition, I needed to carry 6 apples, 20 eggs and 2 of milk. That was equal to approx. 10 of my own mass.

Questions:

1. How far did the person move?

....

2. What was the person carrying?

....

3. How long did it take to get from the grocery store to the house?

....

4. How many kgs of matter in a solid state did the person carry?

....

5. How many kgs of matter in a liquid phase did the person carry?

....

6. What is the mass of the person?

....

7. How did the person move from the grocery store to the house?

....

8. What was the kinetic energy of the person with the load during the movement?

....

If you cannot answer the questions above, decide what should be added in the text to make it clearer and allow you to answer most of the questions above. What additional information do you need to find in external resources? Try to do that to complete the answer to all questions.

Rewrite the story here and answer the questions once again.

....

....

....

....

....

Share and compare your answers with another group. In case of different answers, share the fragment of the story which led you to this answer and compare it with the same fragment rewritten by another group.

What is your group's conclusion drawn from the exercise?

……

……

……

B. (optional) Working in a group, write your own story using as many units as you know are possible. Below the story, write five questions that can be precisely answered on the basis of your story. Give your questions to another group, and after 5 minutes, check if the answers are correct.

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