UNITS, DENSITY, AND CARTESIAN DIVER

STUDENTS INVESTIGATE RELATIONSHIPS BETWEEN MASS, VOLUME, DENSITY, AND BUOYANCY WITH A CARTESIAN DIVER

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SUITABLE FOR AGE(S)	SUBJECT(S)	KEY FOCUS
12- 18 years	Physics	Measurement Unit transformation

INTRODUCTION

The Cartesian diver is a well-known and visually engaging physics toy that illustrates basic concepts of density and buoyancy. It consists of an object with a bubble of air that changes its volume when pressure is applied. As a result, the diver's overall density fluctuates. If the diver is designed with a density slightly lower than that of the liquid, it will float normally, but it sinks if the pressure increases. The Cartesian diver is also used as an illustration for the fish with a fish bladder.

This task is based on inquiry-based learning (IBL), progressing through three levels: structured inquiry, guided inquiry, and open inquiry. The introductory section requires only basic technical skills for assembling a diver and is suitable even for primary school students. The same task could be used in lower secondary school, where students are already familiar with simple measurements of mass, time, length, and temperature, to extend the concept of measurement or as motivation for introducing density. Finally, the Cartesian diver is used after buoyancy is explained to test understanding of both concepts—density and buoyancy—and their connection.

Several real-life connections could be discussed, beyond fish and the density of solutions. Ships are made of iron and do not sink; boats are made of wood, so do they sink? How are the capacities of ships determined?

The task serves not only as a scientific exploration but also as a social tool to facilitate transitions such as establishing relationships with new teachers and peers, and as an interdisciplinary bridge across physics, chemistry, biology, and mathematics.

TASK DESCRIPTION

This rich task is designed to explore how the physical properties of divers affect their ability to float or sink. It introduces the novel property of "floatability" and leads learners through increasingly autonomous investigative steps:

- Part 1 – Structured Inquiry: Students construct simple divers and explore the concept of floatability. The first part introduces the construction of a diver and discusses the "new" property of an object called "floatability", along with the process used to measure it.
- Part 2 – Guided Inquiry: Learners design experiments to examine which properties affect floatability. The second part studies the dependence of floatability on the properties of an object. It is conducted as a guided inquiry using a poster to control variables.

- Part 3 – Open Inquiry: Students construct a Cartesian diver and explore its behavior under pressure. Finally, the Cartesian diver is constructed, and its properties, behaviour, etc., are studied as part of an open inquiry.

Key concepts addressed include measurement, density, buoyancy, and experimental skills such as designing experiments, predicting outcomes, organising data, drawing conclusions and presenting findings.

Additionally, since a diver's properties influence its floating ability, a teacher can guide students through the transition from comparing properties to measuring and defining a new property called "floatability. " When analysing how a diver's properties affect floatability, students realise that basic properties like mass and volume are insufficient to determine when an object will float or sink. This understanding highlights the need for the concept of density. Ultimately, studying how fish swim at various depths offers an engaging interdisciplinary science project.

TASK PREPARATION

The task is simple, and each student can work alone. However, it is better for each student to assemble their own diver, but they work in groups of three or four. Therefore, the materials needed are provided per student, per group, and per class, that is, per teacher.

Materials needed:

Per Student:

- 15–20 paperclips
- 2–3 drinking straws with a diameter big enough that a paperclip can be inserted into the straw (or soft transparent plastic tubes).

Per Group:

- Scissors
- Ruler
- a glass filled with water (at least 15 cm high)

Per Class:

- Aquarium filled with water (for demonstration)
- Teacher's set of materials for demos
- Worksheets
- 10 sticky notes per student/group
- A4-sized inquiry posters (students)
- A1-sized demonstration poster (teacher)

Note: Due to environmental regulations, plastic straws might not be available. As an alternative, paper straws can be used; however, they are suitable for Parts 1 and 2, but not for the Cartesian diver, where the straws become wet. A good alternative is plastic tubes with an inner diameter similar to that of paper clips. These can be purchased by the metre and then cut into the appropriate pieces. Choose tubes that are as flexible as possible with thin walls. They are usually transparent, allowing for the observation of water content during the manipulation of the Cartesian diver. However, they are slightly heavier, so the teacher must find the optimal length for the diver. The diver needs to carry at least five paper clips before it sinks. If you still have plastic straws available, stockpile them for future use (Figure 1).

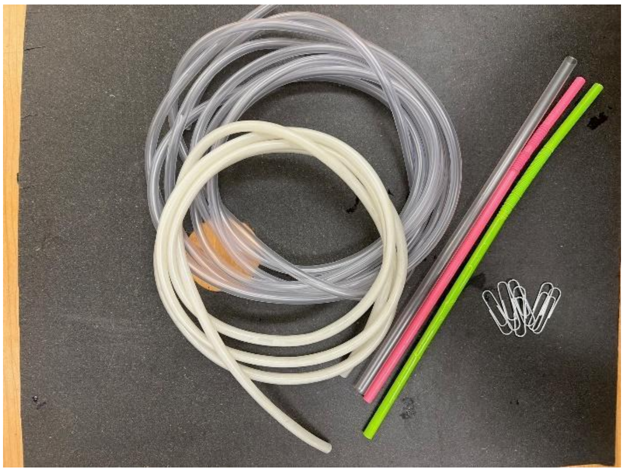

Figure 1. Materials needed for assembling the diver. Soft transparent tubes are a good alternative to plastic straws.

TASK IMPLEMENTATION

The first part could be used when measurements are introduced. Measurements of distance, mass, time, and temperature are covered in lower and upper secondary school, where physics usually begins. These are already familiar to students. Learning the principles of measurement procedures based on methods they already know by heart does not present a challenge. Therefore, "inventing" a new property and designing a measuring method for it, which is independent of the measurer, might be more challenging and motivating.

The second activity can serve as motivation for introducing a new property of an object—its average density—since neither mass nor volume alone can predict an object's "floatability". Investigating and testing floatability can be an engaging activity to accompany the introduction of buoyancy.

Understanding the concept of density is important in chemistry and biology. Since the average density is determined as the ratio of an object's mass to its volume, it can also be utilised in mathematics exercises.

Life in water is an important topic in biology. Understanding the concept of buoyancy helps explain why underwater plants do not need stems but sometimes require roots, and how fishes with a swim bladder or without one maintain their depth. Finally, measurements are typically introduced in the first few hours of physics in lower and upper secondary school, where students encounter new teachers, schools, or peers. The activity involves manual work, planning, and collaboration in general. Therefore, it

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can foster the development of social relations among students and allows the teacher to observe motivation, eagerness, or their absence in new students.

Part 1: Structured Inquiry – "Let Us Make a Diver"

The 10 cm piece of drinking straw is cut and folded in half. Both halves are joined by a paperclip, called paperclip 0 (Figure 2). Four more paperclips are directly attached to paperclip 0. The final object is referred to as a "diver".

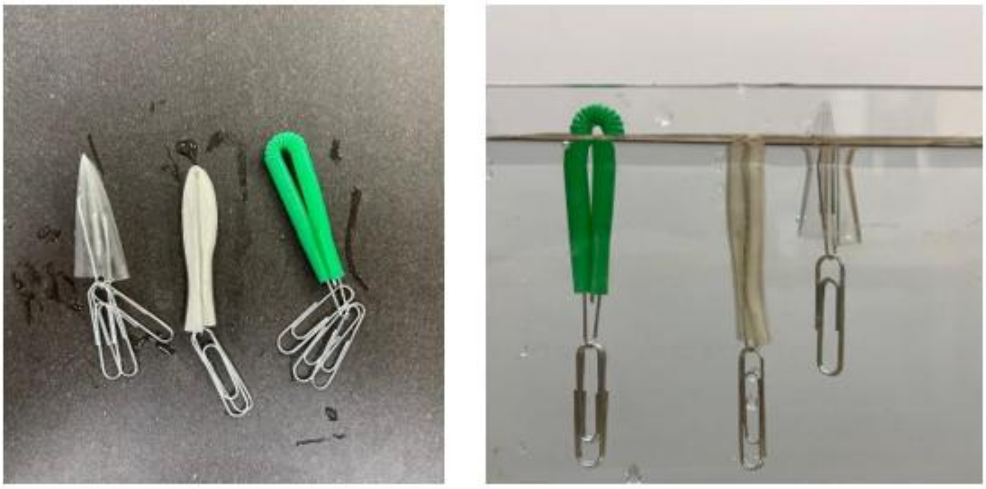

Figure 2. Left: Assembled Cartesian diver, the middle one is made from a rubber tube. Right: Floating Cartesian divers. It is evident that the diver from the rubber tube is heavier. This could be compensated by a longer part which contains air.

Students test whether it floats or sinks by placing it in water. They compare different divers to see if they float or sink, and then they define what it means to "floating well." They also decide on the property they are comparing, such as which diver floats better. The next step is to prepare for introducing a new property of a diver called "floatability." Students find out how many paperclips can be added as a load before the diver sinks and how many can be removed while keeping the diver floating comfortably. Finally, the teacher discusses the measuring procedure:

- Is it possible to define equal "floatability"? How?

- Is it possible to compare floatability as better or worse?

- How can one establish a unit? What could serve as a unit for floatability?

While the first two questions are relatively simple and yield two answers, either the part of the straw above the liquid, or the number of clips (or, for more precise measurements, a mass of a load) that cause a diver to sink. Both easily provide an answer to the first two questions.

To establish a unit, one should choose a standard diver. For example, one could choose a diver that still floats with an upper part perpendicularly to the water and count the number of paperclips needed to sink that diver. The number of paperclips is a measure of floatability. Alternatively, one can measure the height of the straw above the water, but in this case, the measure is applicable for this exact type of diver and cannot be generalised to other objects.

Part 2: Guided Inquiry – "What Affects Floatability of a diver?"

This part is accompanied by the IBL poster. Students receive an A4-sized poster and about ten small sticky notes per person or group. By this stage, students already have a general understanding of floatability.

The teacher poses the question: "Which properties of the diver affect its floatability?" Students write one property on each sticky note and place them under "These may have an influence on".

Students use a second colour of sticky notes to write one observable or measurable property of the diver on each note, according to their means or tools.

The teacher reviews suggestions for variables from the "local" posters and integrates them into the main poster. Students record their choices in notes in their worksheets.

Next, each group decides which variable they will study and moves that variable from "What can we observe/measure" to "We will change/vary". Then they select the property they believe will be affected and move it to "We will observe/measure". All other variables are transferred to "We will not change".

Finally, students formulate a question about how these two variables are interrelated and make an educated prediction or guess, providing an explanation.

These steps help novices in IBL formulate an inquiry question, consider experiments that provide an answer to it, and identify independent, dependent, and control variables, although they are not referred to by these names. As this activity introduces problems that are solved by the introduction of the concept of density, the teacher must ensure that at least one group studies the effect of the length of drinking straws. Students design a testing experiment, which is, in this case, simple. They create a new diver with different properties, choosing from the option "we will change/vary". They then measure its floatability or compare the floating of two divers with different properties.

The teacher should encourage different groups to reach two contradictory conclusions that can both be supported by measurements or observations.

- The heavier the diver, the less floatability.

- The heavier the diver, the higher the floatability.

The latter requires some discussion and applies to divers with the same load but different straw lengths.

This contradiction highlights the need for another property, density, which predicts whether an object will float or sink in a specific liquid. From this point, the teacher can begin lectures on density.

Part 3: Open Inquiry – "The Cartesian Diver"

The teacher asks the students to make a diver with a very small floatability. They usually add a few paperclips, but alternatively, students may construct a new diver with short straws. Any of the two is acceptable.

Fill the PET bottle with water to the top. Put in the diver. Fix the stopper.

Press the bottle from the sides and observe.

If the Cartesian diver is assembled properly, it barely floats without pressing the bottle, and it sinks when pressure is applied by hands (Figure 3).

Figure 3. Left: Assembled Cartesian diver. The diver floats, if the PET bottle is not compressed. Right: Under pressure it sinks.

Students are encouraged to investigate: Why does the diver sink when pressure increases, and why does it float? This part of the activity can serve as motivation before the buoyancy is introduced, but it is suggested that this be revisited after the introduction. The diver is an object that changes shape under pressure. If the pressure is increased, the water is pushed into the straws, and the volume of the air trapped above the water becomes smaller. So far, everything is logical. But to consider the average density of the diver, one must decide which components are included as part of the diver and which are not.

There are two options: either the water in the straws is part of the diver, or it is not. In the first case, the volume of the diver remains constant, and the diver's mass increases with increasing pressure. As a result, the average density of the diver also increases with pressure and eventually becomes equal to the density of water at a specific pressure. When pressure increases further, the diver sinks. Criterion (i) is used in this reasoning. In the second case, the diver consists of solid parts and the air bubble within the straw. As pressure changes, the water level in the straw fluctuates, which causes the diver's volume to change. However, the diver's mass remains constant. The reasoning (ii) is applied here, which also shows that as pressure rises, the diver's volume decreases, resulting in an increased average density. In physics, especially when dealing with quantities defined as ratios, different lines of reasoning often lead to the same conclusion. Therefore, it is wise to direct activities so that different groups consider either changes in the diver's volume or its mass, but arrive at the same conclusion that is consistent with the observation.

Now, several situations can be studied. Several new problems can be posed. For example,

- How does the position of applied pressure affect the behaviour of the diver?
- Can one keep the diver in the middle of the bottle?
- Can one use another object as a diver? Which properties must such an object have?
- How can one make a stable position of the diver in a bottle? Is it even possible?

KEY LEARNINGS

This task has been tested multiple times under various conditions. In person, it was conducted with three large groups of in-service biology and chemistry teachers, as well as with two groups of pre-service physics teachers, both at the University of Ljubljana.

Remotely, it can be adapted for situations where students are present but the teacher works remotely, such as for hybrid settings, if travel for the workshop leader is not feasible. For example, the workshop on the Cartesian diver was conducted with teachers participating in the STAMPEd project training course at the 2022 GIREP conference. Additionally, the unit was tested with lower secondary school students during Researchers Night. The activity typically takes between 60 and 90 minutes. Assembling the Cartesian diver is straightforward enough for younger students in lower secondary school. The cognitive conflict readily engages the youngest group, and understanding the density aspect is also relatively simple for them, as students tend to relate the concept of density to the proportion of air in floating objects.

CONCLUSION

The task presented comprises three parts: structured, guided, and open inquiry. Each part can be used independently. For instance, the first structured part may be employed at the start of the school year when physics units are introduced during one of the initial physics lessons. Since it involves manual and collaborative work, a new physics teacher can observe students' skills, social interactions, and related behaviours.

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If students are new to the programme, such as during the transition from lower to upper secondary school in some systems, it can also serve as an opportunity for socialisation, especially if conducted during one of the first physics lessons, or even the very first.

The second part is a guided inquiry designed as an introductory activity for the concept of density. However, students should already be familiar with the first part, or the teacher must facilitate both parts during the same lesson, either in a 45-minute session or less. The final part can be quick and entertaining, but it also has the potential to develop into a deeper discussion about reasons for sinking and floating, as well as the definitions of what is included in the volume and the mass of the diver.

Given the wide variety of divers, the activity can easily be expanded into a project, analysing different aspects of floatability. Several ideas can be found in references, ranging from various types of divers to divers in different liquids and many other topics.

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APPENDIX

Measurements

Although a measuring procedure is considered trivial for physicists, it is worthwhile to increase awareness about the conditions that must be met for the realisation of a reliable measurement. Usually, five conditions must be fulfilled.

Let us imagine that one wants to design a measuring procedure for a property P of an object O, shortly P(O). Which conditions should the property fulfil?

I. There should exist a procedure which undoubtedly distinguishes between

P(O1) = P(O2) and P(O1) ≠P(O2)

II. If P(O1) ≠P(O2), there should exist a procedure which undoubtedly distinguishes between

P(O1) > P(O2) and P(O1) < P(O2)

III. One has to be able to determine the unit, that is, the standard object that has a property P(O) = 1 unit.

IV. One has to be able to determine the procedure, how to combine standard objects to achieve

P(O combined) = P(O)

and a rule on how to form a number from this combination. The procedure is called "concatenation".

V. Finally, the same properties of standard objects and the measured object are rarely directly compared/equalised. Usually, another property of the measuring equipment is used, which is easier or more accurately measured. However, one must be sure that values obtained by direct measurements of the property and values deduced from another property are the same. The procedure to ensure that is called "measuring transformation". For example, length could be deduced from the time of travelling of a signal, voltage can be read from the angle of the voltmeter's hand, etc.

The procedure is usually taken for granted, but if measuring a new property is introduced, students may become aware that a procedure for measuring a property is usually well considered and long tested. The teacher can also use this activity as an alternative to boring, well-known measurements of length, mass, etc., as one of the introductory lectures to physics. There are many other properties that call for "invention" of measuring procedures, for example, absorption of tissues, fragility of spaghetti, elasticity of apple peels, value for money for oranges, etc.

Density

Density is a property of a material and is an extensive property as it may change from one point to another. In contrast, an average density is a property of an object; however, it is still not an additive property. When two objects are combined, the average density can increase, decrease, or remain the same.

Density ρ as a property of location is defined as a ratio between a small mass dm that is found in a small volume dV around the point in space r

$$\rho(r)=dm/dV \quad (1)$$

or as the average density of an object

$$\rho=m/V \quad (2)$$

where ρ is the average density of the object, m is the mass of the whole object, including all its components, and V is the volume of the whole object.

Direct comparisons of the densities of two objects can be done in two ways.

- If both objects have the same volume, the object with a larger mass, has a higher density.
- If both objects have the same mass, the object with a smaller volume has the higher density.

Comparison of density that considers one or both former statements is easily realised with liquids and granular materials. Pouring the material to the container, one can almost continuously change either the volume or the mass of an “object” if the object is considered as a content of the container. However, for objects in general, this is not possible. An alternative method for comparing densities of objects is available when objects are submerged in liquids. For floating objects, the weight of the object is equal to buoyancy, the density of the object and the liquid can be directly compared using the reasoning in (ii), and different floating objects are compared by the shares of volume above the liquid. The method is very good for observation and reasoning, but not for real measurements, except when a standard object is used and the density of a liquid is measured, such as with an alcoholmeter. To obtain a numerical value for density, Eq. (2) is used, and the density is calculated from the known volume and mass.

Determining the average density of an object can become complicated if the object does not have either a constant volume or a constant mass. Therefore, deciding which components are a part of the object, and which are not, is crucial, and activities that raise awareness about that are welcome.

Buoyancy

Buoyancy is a force that acts from the surrounding liquid (and/or gas) on an object immersed in it. Following Archimedes, its magnitude is equal to the weight of liquid displaced by an immersed object, with direction opposite to gravity. In some cases, it is generalised, for example, in rotating systems. Rarely is buoyancy called a force that is the sum of all partial forces due to hydrostatic pressure. The resulting force caused by hydrostatic pressure is a reason for buoyancy, but this last meaning could entirely change the concept of buoyancy. For example, buoyancy is the force that holds a vacuum hook on a ceramic tile. We will not use this last meaning in this contribution, but will stick with the original Archimedes’ meaning.

Let us now form a criterion for floating and sinking:

- If the weight of the object is larger than the weight of the same volume of liquid, not only is the liquid displaced, but also the whole volume of the object; the object sinks.

- If the weight of the object is smaller than the weight of the same volume of liquid, the object floats, partially above the water.

- There is no third option in practice, except if one considers floating of the (same) liquid in a liquid. There are always slight differences in those two weights, resulting in the object either sinking very slowly or rising to the surface if it is immersed entirely, as is typical for plankton, or an active control of its position in the liquid, as submarines and fish do.

Here, we immediately notice how important it is to decide what constitutes a part of the object and contributes to its volume/mass, and what does not.

Now, we compare two objects with the same volume (the object and the liquid), one can easily extend this comparison to well-known and usually used direct comparisons of densities.

The task, which was carefully planned into three parts, addressed the three different issues discussed above. The first part of the activity changes the mass, the second part changes the volume but has a negligible influence on the mass, and the third part, which involves the actual assembly of the Cartesian diver, considers both. When the pressure in the bottle is increased, the water enters the straw. If the straw with its full content is included as a mass of the Cartesian diver, its volume is constant, but mass increases with more water in the straw. The average density of the Cartesian diver also increases. On the other hand, one can consider the water, even when it enters the straw as a part of the surrounding, then the mass of the diver is constant, but the volume varies and decreases with applied pressure. Again, the average density increases and eventually the diver sinks.